

*Adaptation by Xavier Adserà of  
Excerpt, Chapter 14: "Sensitivity Analysis for Company Valuation: the Discount Rate" in  
"Principios de Valoración de Empresas"  
Xavier Adserà & Pere Viñolas  
Ed. Deusto - 1997. Print.*

## SENSITIVITY ANALYSIS FOR COMPANY VALUATION: THE DISCOUNT RATE

This chapter will analyse a company's value sensitivity to changes in the discount rate. Starting with the premise that the discount rate is a combination of the risk-free rate and the risk premium specific to an individual company, changes originated by variation in either the risk-free rate or the risk premium component will have a similar effect on the overall discount rate. The main drivers of the discount rate (and by extension of its risk components) are interest rates. The other key point to highlight is the fact that the discount rate required by investors is equivalent to a company's cost of capital (net of issuance costs) or required rate of return by investors, and thus the terms are often used interchangeably. Throughout this chapter, we will endeavour to lay out the underlying principles behind the drivers of value Sensitivity to changes in the discount rate, placing particular emphasis on the role of required returns by investors and expected growth rate.

### 1. VALUATION SENSITIVITY TO CHANGES IN THE INTEREST RATE

Despite the lack of academic literature on the effect of changes in the discount rate on the fair value of an individual company's stock, a unanimous consensus among analysts and empirical evidence from the financial markets suggest that the relationship should be negative. In other words, an increase in the discount rate will imply a decrease in the share value and vice versa.

This article aims to find the basis of said relationship: what makes one company's shares more volatile to changes in the discount rate than another? What determines the sensitivity to these changes? It is clear that share value changes with these variation, but by how much?

#### 1.1 The Concept of Asset Duration

Duration measures interest rate risk and thus is a prominent metric used in fixed income analysis. Named "Macaulay Duration" after its original author, it measures how long it would take to recoup the discounted cash flows for a project at a market discount rate in a number of years. In other words, duration measures the average life of the present value of the expected cash flows.

The duration formula is as follows:

$$D = \frac{\sum_{s=1}^n \frac{s \times C_s}{(1+I)^s}}{P}$$

where:

D = Macaulay's Duration,  
P = asset price,  
s = time period in years,  
C<sub>s</sub> = time period's cash flow  
n = total number of time periods  
I = asset's internal rate of return at the time of the computation

From this formula we can conclude that the duration value determinants are as follows:

1. Investment period: the longer the cash flows time frame, the greater the duration
2. Interest rate: the greater the rate, the shorter the duration
3. Term structure of cash flows: the greater the closest cash flows, the shorter duration and vice versa

The usefulness of duration relies on its ability to compare different assets from an interest rate risk perspective. To make this concept clearer, we will use an example with two different projects. These projects present certainty of cash flows (zero variation probability) and no counterparty (default) risk. These assumptions allow us to use the risk free rate as the required return for both, which we set at 10%.

The returns for each project are as follows:

- Project A: investor receives £110m one year after the time of investment. There are no additional inflows after that
- Project B: investor receives £10m at the end of every year after the time of investment for an infinite number of years (£10m constant perpetuity)

To compare both alternatives, we would discount the cash flows to the present time using a 10% interest rate, which would result in both projects valued at £100m. We would then analyse the effect of a variation in the interest rate, used as the discount rate, in each project. We would therefore conclude that Project B is riskier as it presents a higher standard deviation. However, how much riskier is it and why?

In order to provide an answer to these questions, we must compute the duration of both projects to measure their intrinsic interest rate risk<sup>1</sup>:

- Project A duration: 1 year
- Project B duration: 11 years

The project with a longer duration will have the greatest risk given interest rate movements. Also notice that, despite its infinite nature, Project B presents an equivalent duration of 11 years. This means that Project B is financially equivalent to a 11-year zero coupon bond.

Using the duration formula, we can compute the price elasticity to changes in the interest rate. This is known as "Adjusted Duration" or "Sensitivity"<sup>2</sup>:

$$\frac{\Delta P}{P} = -\frac{D}{(1+I)} \times \Delta I$$

The quotient  $\frac{D}{(1+I)}$  corresponds to Sensitivity, therefore:

$$D^* = \frac{D}{(1+I)}$$

We just have to replace:

$$\frac{\Delta P}{P} = -D^* \times \Delta I$$

<sup>1</sup> Project A will have a duration of 1 year as it only presents one cash flow in year one. Duration can be defined as the time to maturity in an equivalent zero coupon bond, which in this case is 1. For project B, its duration can be found using the constant perpetuity simplified form  $\frac{Cs \times (1+I)}{I}$ .

<sup>2</sup> Despite a lack of consensus in the field, the terms "adjusted Duration" and "Sensitivity" are not exactly equal. While the adjusted Duration expresses changes in asset value as a percentage, Sensitivity represents the absolute change in the price of the asset in terms of monetary units when interest rates vary. In order to simplify, both terms are used interchangeably.

Then, we can find the duration from the Sensitivity. Back to our example:

*Project A*

$$P = \frac{110}{(1+I)}$$

$$D^* = \frac{\partial P}{\partial I} \times \frac{1}{P} = \frac{-110}{(1+I)^2} \times \frac{1}{P}$$

because  $I = 10\%$  and  $P = 100$ , then  $D^* = -.9091$

$$D = -D^* \times (1+I) = .90910 \times 1.1 = 1$$

*Project B*

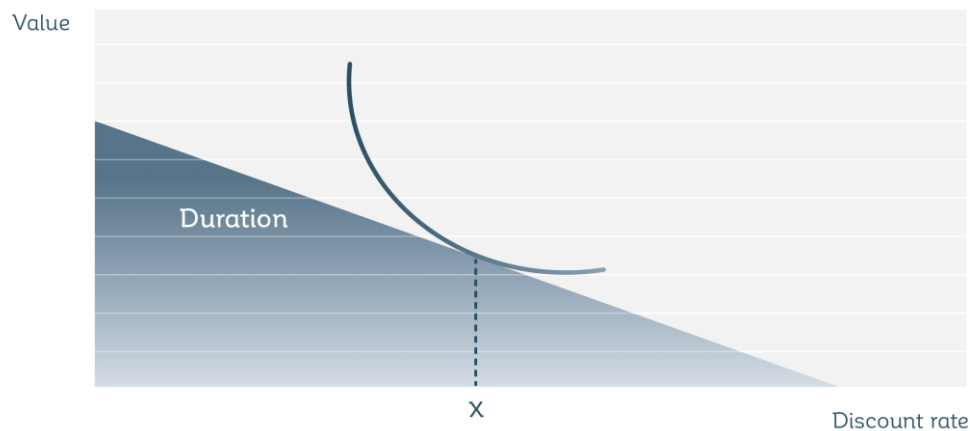
$$P = \frac{10}{I}$$

$$D^* = \frac{\partial P}{\partial I} = -\frac{10}{I^2} \times \frac{1}{P}$$

because  $I = 10\%$  and  $P = 100$ , then  $D^* = -10$

$$D = -D^* \times (1+I) = 10 \times 1.1 = 11$$

GRAPH 38



Graph 38 illustrates the relationship between an asset's discount rate and its present value as defined by the adjusted duration formula (straight line) and the slightly more sophisticated equation depicting the true behaviour of the function, which includes a convexity factor (curved line)

The relationship between asset value and interest rate in computing the present value is not a linear one, but slightly convex. The Sensitivity is a linear approximation to this function.

The partial derivative of the function depicted on graph 38 measures asset value Sensitivity to changes in the interest rate. Consequently, the adjusted duration represents the slope of the tangent to the curve at a given point, which is the internal rate of return at the moment of the calculation. As it can be extracted from the graph, due to the curve's shape, when the interest rate change is small, the adjusted duration provides a good estimation.

Likewise, and due to the linear nature of duration, the greater the changes in the interest rate, the more it will diverge from the curve. This divergence will increase with convexity, which measures the degree of curvature.

Another term we will be using is *portfolio duration*, defined as the weighted duration of a portfolio containing more than one asset of similar nature.

## 2. CONVEXITY

Convexity measures the degree of curvature in the relationship between an asset's value and the applied discount rate. Its calculation refers to the variation of an asset's sensitivity to changes in the discount rate. In other words, it is the adjusted duration partial derivative with respect to the discount rate, or the second derivative of the price with respect to the discount rate.

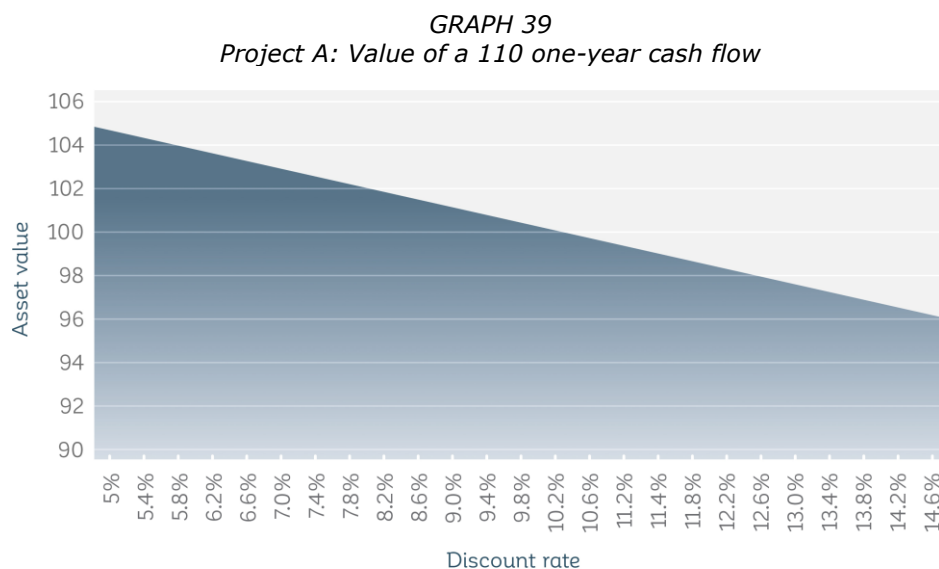
Both duration and convexity are well-established concepts in fixed income analysis and management, but seldom used when analysing the value of equity<sup>3</sup>.

As previously mentioned, duration will be a good estimator of the variability of the price to changes in the discount rate as long as:

- These variations are small and/or
- The curvature of the function is not pronounced

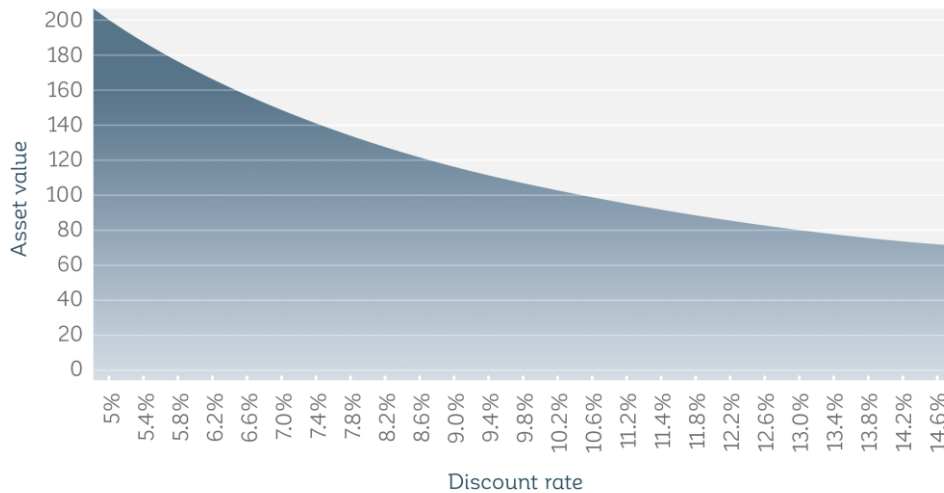
Explanatory graphs on the relationship between asset value and the discount rate applied to the aforementioned projects can be seen below. As shown, Project B has a greater curvature than Project A.

From the graphs, we can also observe that Project B presents a higher convexity than Project A. Consequently, its adjusted duration will be a much better estimator of variations on the asset's value to changes in the discount rate than for project B.



<sup>3</sup> Lewis D. Johnson: "Convexity for equity securities: does curvature matter?". Financial Analyst Journal, Sep-Oct 1990.

GRAPH 40  
Project B: Value of a perpetuity with payment equal to 10



Convexity implies that the value of the asset will increase more than proportionally against a decrease in the discount rate and it will decrease less than proportionally against an increase in the rate. This means it can be used as downside protection while increasing revenue potential. A simple sensitivity analysis will not take such effect under consideration, overestimating the decrease of an asset's value as a result of an increase in the risk free rate, and underestimating the increase caused by a reduction of this same rate.

We can also extract from the graphs that this effect is greater in Project B than A due to the former's higher convexity.

Taking the first two terms of Taylor series, we can develop an approximation to the curvature that relates the price and the required rate:

$$dP = \frac{\partial P}{\partial k} \times dk + \frac{1}{2} \times \frac{\partial^2 P}{\partial k^2} \times (dk)^2 + Error$$

Dividing all the parameters for the asset's value:

$$\frac{dP}{P} = -Adjusted\ duration \times dk + \frac{1}{2} convexity \times (dk)^2 + Error^4$$

Then, we will compute the approximation applied to the examples we have been working on:

*Project A:* Cash flow value after one year:

$$P = \frac{110}{(1+I)}$$

$$\frac{dP}{P} = -\frac{1}{(1+k)} dk + \frac{1}{(1+k)^2} (dk)^2$$

*Project B:* Value of a constant perpetuity:

$$P = \frac{10}{k}$$

<sup>4</sup> As it has been mentioned, the adjusted Duration, also called Sensitivity, is found by dividing the asset's Duration by (1+k). ½ convexity is also known as "simplified convexity".

$$\frac{dP}{P} = -\frac{1}{k}dk + \frac{1}{k^2}(dk)^2$$

We can conclude that for both Project A, a zero-coupon asset, and Project B, a constant perpetuity, both the Sensitivity to changes in the interest rates and the asset convexity will increase when the discount rate (k) decreases.

The table below shows how the different variables would behave when a 1% change in the discount rate is applied to our 10% base case scenario.

TABLE 13  
Project A: Cash flow value after one year

Rate	Price	dp/P	dp/P e. sens.	1 <sup>st</sup> degree difference	Convexity	Estimated P	2 <sup>nd</sup> degree difference
9%	100.917431	.9174%	.9091%	.0083%	.0083%	.9174%	.0001%
11%	99.099099	-.9009%	-.9091%	.0082%	.0083%	-.9008%	-.0001%

Project B: Constant perpetuity value

Rate	Price	dp/P	dp/P e. sens.	1 <sup>st</sup> degree difference	Convexity	Estimated P	2 <sup>nd</sup> degree difference
9%	111.111111	11.1111%	10%	1.1111%	1%	11%	.1111%
11%	99.909091	-9.0909%	-10%	.9091%	1%	-9%	-.0909%

The first column takes into account the Sensitivity or adjusted duration resulting in the first degree difference between the real variation and the one estimated using the adjusted duration formula.

If we use a 9% discount rate in Project A, the asset value increases to 100.917. This results in a variation from the original value of .9174%. On the other hand, the adjusted duration concluded that the variation should have been +.909%. The first difference or divergence between the estimated and original variation is .0083% (.9174% - .9091%).

For Project B, a 1% discount rate decrease means an appreciation in the asset value to 111.111, translated into a 11.111% increase. The adjusted duration returns a 10% variation. We can then conclude that the first difference is 1.1111% (11.1111% - 10%).

The difference between the adjusted duration estimate and the observed variation can be explained through convexity. Because Project A has smaller convexity, the adjusted duration estimation is very precise. Project B however, has a higher discrepancy due to its higher convexity. We reach the same conclusion for a variation of a 1% increase in the discount rate.

After this first approach, a second in depth analysis that includes convexity has been carried out. As we have mentioned, this is not completely necessary for Project A, but it is for B. From this second

estimation, we compute the difference with respect to the real variation. This results in the second difference, which measures the error factor described by Taylor.

Using the first two terms in Taylor's series, adjusted duration and convexity, we can further refine our estimate for the real variation in Project A as the second degree difference is very small. On the contrary, in Project B these parameters are higher, resulting in a less accurate result.

It is also worth mentioning that convexity behaves inversely to duration and cushions its impact.

Moreover, despite large changes in the discount rate, such as a 1% variation, the error percentage in the second degree difference is small in both cases. Smaller variations will obviously imply better estimations.

Up until now we have analysed an example with only one cash flow and another with constant cash flows (albeit a constant perpetuity). In the following section, we will expand our analysis to include cash flow growth potential as a variable.

### 3. DURATION, CONVEXITY, AND GROWTH RATE

In order to explain the relationship between convexity and growth rate, we will begin by reviewing the Gordon-Shapiro asset valuation model. According to this method, an asset's present value is equal to the present value of a perpetuity where its cash flows (dividends) grow at a constant rate from period to period, starting with the first year's expected dividend ( $d_1$ ) and using  $k$  as the required discount rate and  $g$  as the expected growth rate:

$$P = \frac{d_1}{k - g}$$

Using Taylor's model:

$$dP = \frac{\partial P}{\partial k} \times dk + \frac{1}{2} \times \frac{\partial^2 P}{\partial k^2} \times (dk)^2 + Error$$

Dividing all terms by  $P$ :

$$\frac{dP}{P} = \frac{\frac{\partial P}{\partial k}}{P} \times dk + \frac{1}{2} \frac{\frac{\partial^2 P}{\partial k^2}}{P} \times (dk)^2 + Error$$

Computing the derivatives:

$$\frac{\partial P}{\partial k} = -\frac{D}{(k - g)^2}$$

$$\frac{\partial^2 P}{\partial k^2} = -\frac{2D}{(k - g)^3}$$

We now just have to divide these derivatives by  $P = \frac{D}{k - g}$  and expand the equation:

$$\frac{dP}{P} = -\frac{1}{k - g} \times dk + \frac{1}{(k - g)^2} \times (dk)^2 + \varepsilon$$

In the meantime, we have also derived the general formulation for duration and convexity as follows<sup>5</sup>:

$$\text{Adjusted Duration} = -\frac{1}{k-g}$$

$$\text{Convexity} = \frac{1}{(k-g)^2}$$

Please note that, in this example, Project A is assumed to have a  $g$  equal to 0 given that cash flows are assumed to start growing after the first period.

Continuing with our example, we now examine what would happen if Project B were to grow at 2% and 4% per annum respectively. Table 14 shows how this rate affects the price in each of the 1% interest change scenarios.

Graphs 41 and 42 likewise present the asset value when the required rate of return goes from 5% to 15%, considering a 2% or 4% growing rate.

**TABLE 14**  
*Project B: Present value of a perpetuity with constant growth of 2%*

		CF	110		Sensitivity	-0.9091	
		Current rate	10%		Convexity	0.8264	
		Price	100				
Rate	Price	dp/P	dp/P e. sens.	1 <sup>st</sup> degree difference	Convexity	Estimated P	2 <sup>nd</sup> degree difference
9%	142.9	14.29%	12.50%	1.79%	1.56%	14.06%	0.22%
11%	111.1	-11.11%	-12.50%	1.39%	1.56%	-10.94%	-0.17%

*Project B: Present value of a perpetuity with constant growth of 4%*

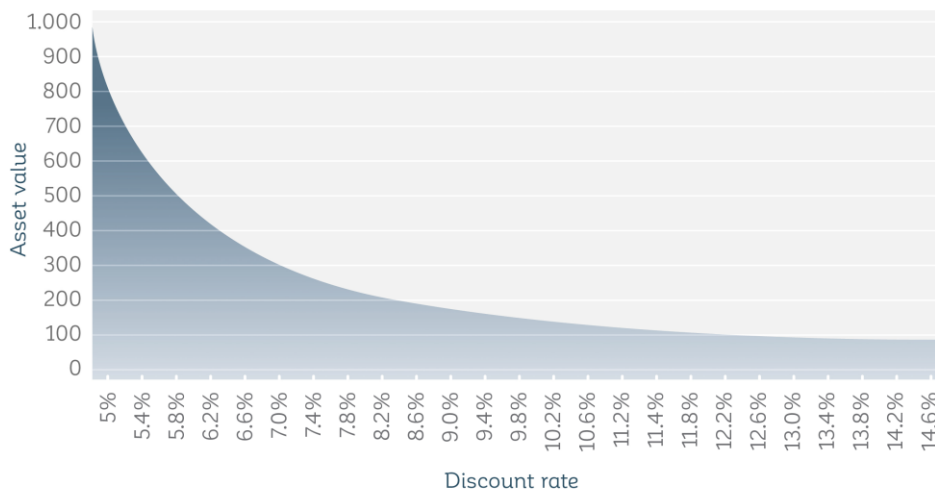
		CF	10		Sensitivity	-10	
		Current rate	10%		Convexity	100	
		Price	100				
Rate	Price	dp/P	dp/P e. sens.	1 <sup>st</sup> degree difference	Convexity	Estimated P	2 <sup>nd</sup> degree difference
9%	200	20%	16.67%	3.33%	2.78%	19.44%	0.56%
11%	142.9	-14.29%	-16.67%	2.38%	2.78%	-13.89%	-0.40%

<sup>5</sup> Note that adjusted duration or Sensitivity equals  $-\frac{1}{k-g}$ , the Macaulay's Duration, expressed in years, applied to a constant growth action is, by definition, equal to:  $D = \frac{1+k}{k-g}$



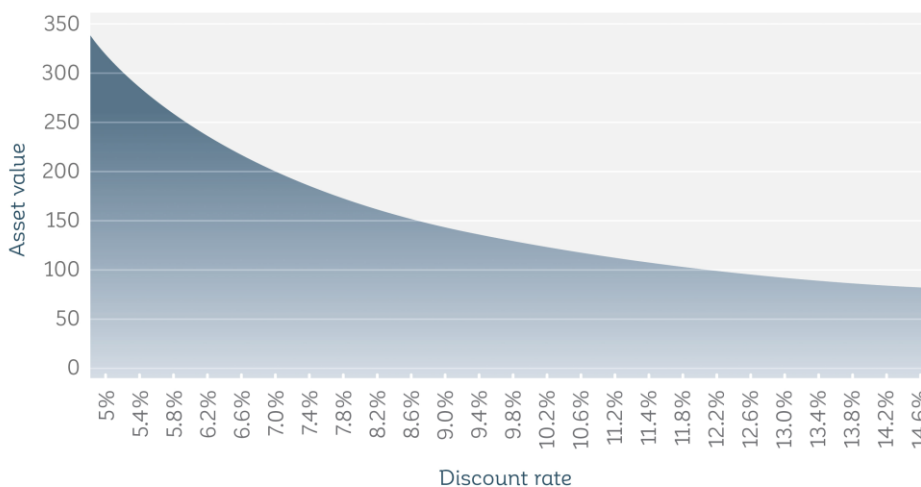
GRAPH 41

Project B: Value of a perpetuity with first term equal to 10 and a 4% growth rate



GRAPH 42

Project B: Value of a perpetuity with first term equal to 10 and a 2% growth rate



We have worked on Project B, a perpetual annuity with an initial cash flow in year 1 (initial period) of 10, in three different scenarios. In the first one, we considered a constant income ( $g = 0$ ). Then, we have used growing rates of  $g = 2\%$  and  $g = 4\%$ . In doing so, we have come to the following conclusions:

The asset adjusted duration increases when the expected growth rate increases. In the example above,  $\Delta 1\%$  in the discount rate causes a 10% ( $g = 0\%$ ), 12.5% ( $g = 2\%$ ) and 16.67% ( $g = 4\%$ ) price variation respectively. Thus we can conclude that the asset value appreciates proportionally with increases in the expected growth rate (growth is positively correlated with duration). These results should not come as a surprise, as an increase in the growth rate will cause farther in the future cash flows to increase exponentially, increasing their weight on present value and thus yielding a higher duration.

A direct relationship between changes in the expected growth rate and adjusted duration are in line with the general formula we derived earlier on, which gave us that adjusted duration is equal to  $\frac{1}{k-g}$ .

The same holds true for duration, given that:

$$D = \frac{1+k}{k-g}$$

Given the nature of duration, the smaller the discount rate (k), the greater the adjusted duration will be.

From a different perspective, the smaller the discount rate, the greater farther in the future flows will be, which increases the duration of the project. In other words, the adjusted duration will grow with a reduction in the parameter (k-g).

Convexity behaves in the same fashion. If we take our example, we can see how with a similar discount rate, convexity will increase in the same way as growth rate. Project B's convexity (variation of duration) to a 1% change in the discount rate implies a 1% (g = 0%), 1.56% (g = 2%) and 2.78% (g = 4%) price variation. This progression will continue as we input higher growth rates.

Looking at the graphs, we can observe the increase in convexity. If we compare Project B without a growth rate, to the 2% and 4% examples, we notice how the curvature steepens as g increases.

The first difference in the example, price variation not related to the adjusted duration, increases when the growth rate rises and, as a consequence, so does the convexity. The convexity adjustment factor is thus also higher.

We can also see this effect proved in the graphs. The smaller the discount rate, the higher the convexity of the curve will be.

Just like with adjusted duration, convexity is also determined by the discount rate and the growth rate. Its general form simplifies to  $\frac{1}{(k-g)^2}$ . Convexity will therefore be higher when the difference (k-g) decreases.

From the above we establish that the two main variables that affect a company's sensitivity to changes in its discount rate are its cost of capital (k) and growth rate (g) with cost of capital moving in the same direction as Sensitivity (direct relationship) while growth rate moves inversely (inverse relationship). Moreover, and given the assumptions and formulaic methodology set forth above, we can conclude that low risk companies with high expected growth rates (low k, high g) are more sensitive to changes in the discount rate, while high risk companies with slow growth expectations (high k, low g) are less affected.

#### 4. DURATION AND THE PRICE TO EARNINGS RATIO

The value sensitivity of an asset to changes in the discount rate depends on the level of the discount and the growth rate.

The dividend discount model shows:

$$P = \frac{d_1}{k - g}$$

The dividend can also be split into current net income and pay-out ratio<sup>6</sup>:

$$P = \frac{\text{Net Income} \times PO}{(k - g)}$$

Re-arranging the formula:

$$\frac{P}{NI} = P/E = \frac{PO}{k - g}$$

Because Sensitivity, or adjusted duration, is equal to:

$$D^* = \frac{1}{k - g}$$

We can conclude that:

$$P/E = D^* \times PO$$

or:

$$D^* = \frac{P/E}{PO}$$

Because of it, we can conclude that the P/E ratio, adjusted through PO<sup>7</sup>, is a good estimator of value sensitivity to changes in the discount rate estimator.

P/E ratio is the result of including the following to the valuation:

- a) investment risk, as the required rate (k)
- b) growth (g)
- c) quality of the growth, as the PO, which shows the required percentage that must be reinvested in order to grow

Both convexity and duration depend on the required rate of return and the expected growth rate. Consequently, and adjusting for growth quality, the duration can be approximated through the P/E ratio. What this effectively means is that companies with a high P/E ratio, whether as a result of low required return or high expected growth irrespectively, will present high Durations. On the other hand, low P/E companies (high risk or low growth expectations), will show low Duration.

<sup>6</sup> By pay-out we understand the potentially distributed profit's percentage without changing the company's financial structure. As we explained in previous chapters, the dividend is equal to the free cash flow.

<sup>7</sup> PO will always be equal or smaller than 1, which results in Sensitivity being higher than the P/E, except cases in where working capital is negative.

## 5. FROM ASSET DURATION TO EQUITY DURATION

Up until now, we have heard about a company's shares value sensitivity to changes in the discount rate using the dividend discount model. However, what is the source of this duration?

As we have seen, duration is determined by the cash flow term structure, which means it's a direct function of the assets held generating said cash flows. This is a concept pertinent to the asset side of the balance sheet.

How do we, then, reconcile the relationship between the required rate of return on equity by investors with changes to the present value of our assets (the magnitude of which is measured by duration)?

The answer resides in the fundamental nature of equity as the residual value of net assets (assets – liabilities). Therefore, any interest risk borne from the asset side, which is not offset by the structure of the respective liabilities held, will directly translate into changes in equity value.

In order to establish a relationship between asset duration ( $D_a$ ) and equity duration ( $D_e$ ), we will use their respective sensitivities or adjusted durations as our starting point. To simplify, we will assume that one investor acquires all the shares and all the debt in the company. Given that the investor owns the entire company (including all of its assets, liabilities and equity), we can be certain the investor also bears all of the risk generated from changes in the discount rate irrespective of their origin.

Remember that a portfolio's adjusted duration equals the weighted average of the durations of each of the portfolio's component which can be formulated as:

$$D^*_a = \frac{E}{E+D} \times D^*_e + \frac{D}{E+D} \times D^*_d$$

$D^*_e$  = adjusted duration (Sensitivity) of equity

$D^*_d$  = adjusted duration (Sensitivity) of debt (liabilities)

$D^*_a$  = adjusted duration (Sensitivity) of assets (changes in EBITDA)

Asset sensitivity defines the risk of changes in the discount rate that come from the cash flow structure generated from the company's activity (EBITDA), which is independent to how those assets are financed (D/E split or chosen capital structure).

Therefore, the adjusted duration of a company's assets will be the weighted average of each of the company's activities. Applying some algebra, the previous formula results in:

$$D^*_e = D^*_a + \frac{D}{E} \times (D^*_a - D^*_d)$$

The flows represented in the balance sheet will determine the sensitivity of our equity<sup>8</sup>. This means that it will be equal to the assets' sensitivity plus/minus the risk coming from the chosen capital structure. The risk derived from changes in the discount rate will behave in an equal fashion as the underlying business risk (EBITDA risk or asset risk).

When talking about duration or sensitivity of debt, we refer to the effect of changes in the underlying interest rate on the market value of such debt. We should not mistake this with the tenure of the debt, as long dated instruments can have very short durations in some cases<sup>9</sup>.

In summary, if the duration of the debt is greater than that of its associated assets, it will absorb risk, if the contrary is true it will magnify it. We can thus conclude that greater leverage will translate into smaller equity duration. On the other hand, if the sensitivity of debt is lower than the sensitivity of assets, the use of leverage will increase our equity exposure to changes in the discount rate.

<sup>8</sup> Sensitivity, or adjusted duration, equals durations expressed in years divided by factor  $(1 + \text{discount rate})$ . If we wanted to formulate this expression:  $D_e = \left[ \frac{D_a}{1+WACC} + \frac{D}{E} \times \left( \frac{D_a}{1+WACC} - \frac{D_d}{1+K_d} \right) \right] \times (1 + K_e)$  taking the WACC,  $K_d$ , and  $K_e$

<sup>9</sup> As an example, a long-term loan with a variable interest rate will present duration very close to zero.

## 6. EQUITY DURATION PARADOX

As previously described, the equity adjusted duration or sensitivity, as computed by the dividend discount model is equal to  $-\frac{1}{k-g}$ . Therefore, Macaulay's Duration, which expresses such Sensitivity in years, can be formulated as:

$$D = \frac{1+k}{k-g}$$

As way of example, Table 40 shows how the duration value of assets changes with variations in the discount rate (k) from 12% to 20%, and a growth rate (g) that goes from zero to 6%:

TABLE 15  
Duration of an asset

k as a %	g = 0%	g = 4%	g = 6%
12%	9.33	14.00	18.67
14%	8.14	11.40	14.25
16%	7.25	9.67	11.60
18%	6.56	8.43	9.83
20%	6.00	7.50	8.57

The paradox reflects that, while the theoretical results show that the duration should fall between 7 and 19 years under nominal growth ranging from 4% to 6%, empirical analysis return values of 2 to 6 years<sup>10</sup>. The answer to this paradox lies on the relationship that necessarily exists between the evolution of interest rates and inflation, and on the relationship between inflation and expected future returns.

In order to examine that relationship, let us break down the value of equity using the "franchise valuation model" which is formulated as follows:

$$E = \frac{\pi^o}{k} + PVGO$$

$E$  = equity value

$\pi^o$  = current profit

$PVGO$  = present value of future growth opportunities

If we assume that returns on new investments will only ever equal the required rate of return (k), it means we also necessarily assume that there are no barriers to entry that will enable us to maintain returns over the cost of capital in the long term. In that case, PVGO equals zero, as the present value of new investments does not add value to the company. In this situation, we can model our equity value as the present value of a constant annual perpetuity:

$$E = \frac{NI}{k}$$

Note that this does not mean that the company's EBITDA is not going to grow, but rather that this growth will not add value, as its PV is 0. Under this circumstance, the equity duration will be equivalent to that reflected in column g=0% on Table 15, which means that, for the discount rate range in the example, the equivalent duration would be fall between 6 and 9 years, bringing us closer in line with the observed empirical results.

Another distorting factor to take into consideration is the fact that the model thus far developed assumes the term structure of interest rates and asset returns are independent of each other, when in

<sup>10</sup> Martin L. LeiPowitz, Stanley Kogelan: "Resolving the equity duration paradox". Financial Analyst Journal, jan-feb 1993. [See original citation]

reality, they are not.<sup>11</sup> If we consider the fact that expected inflation is known to be a key driver of monetary policy and thus drives changes in interest rates (which are a key component to discount rates), we can conclude that changes in the required rate of return by investors are, in part, driven by changes in inflation expectations. Once we account for the correlation and causality factors between interest rates, inflation, asset returns and growth; asset value Sensitivity to changes in the interest rate decreases. (Bringing us further in line with empirical evidence described earlier)

At the end of the day, if the above correction factor brought our regression coefficient to 1, the duration of assets would be zero, with changes in the interest rate (driven by inflation) translating 1 to 1 into equivalent increases in the nominal required rate of return by investors ( $k$ ) which would, in turn, leave the value of our shares unchanged.

The above assertion hinges on the premise that the return on new investments will only ever be equal to the required return. Additionally, we must bear in mind that changes in the interest rate can, to some extent, differ from changes in expected inflation (as one is set by central banks and the other is based on macroeconomic data). Lastly, it is not necessarily correct to affirm that inflation can always be compensated for equivalent increases in the required rate of return, as seen in recent history loose monetary policy has long failed to increase inflation. Because of these inefficiencies, the correlation between the aforementioned variables is less than 1, meaning correcting for them will decrease our computed theoretical duration, but it will not completely eliminate it.

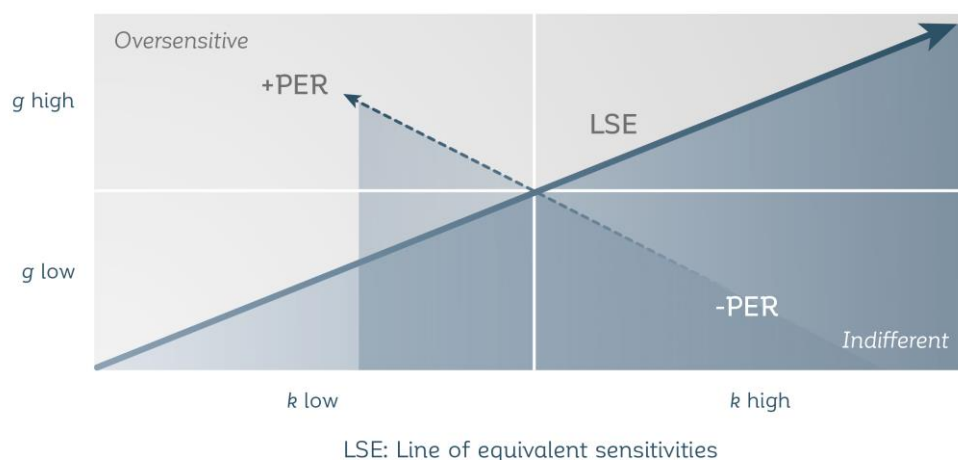
## 7. COMPANY VALUATION AND SENSITIVITY ANALYSIS

As demonstrated in previous sections, sensitivity analysis has important implications for company valuation with changes in the prevailing discount rate directly affecting a company's value.

The first application of sensitivity analysis in equity valuation is built into the relationship between the discount rate and growth opportunities. Looking at companies exhibiting different combinations of the two, we can assign them different profiles along the value matrix depending on their relative reaction to changes in the prevailing required rate of return.

This principle can be illustrated using the following conceptual matrix:

GRAPH 43



<sup>11</sup>As previously outlined in this chapter, the derivative of equity with respect to the discount rate from which we obtain the duration's formula considers that the dividend, ( $Div. = n \times PO$ ) and, as a consequence, the profit, are independent from the discount rate ( $k$ ).

Applying the *Boston Consulting Group matrix for company profiling* to our sensitivity analysis we realise that:

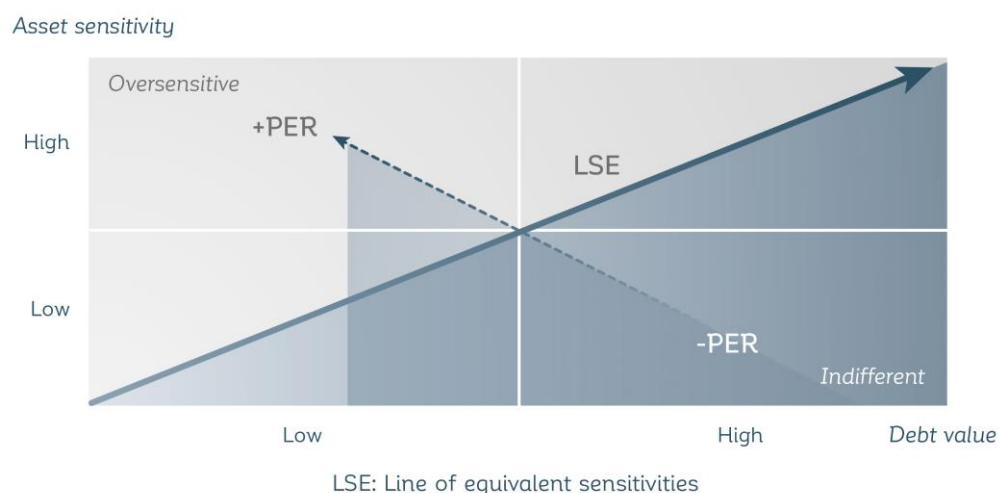
- High risk and high growth companies (*Stars*) do not necessarily imply greater sensitivity to interest rates than mature companies, which present low risk and moderate expected growth (*Cash Cows*). Both profiles can be placed at any point on the ESL and exhibit similar changes in value against changes in the discount rates.
- However paradoxical, companies with high growth expectation but low risk (low  $k$ ), with high quality products and strong underpinning markets (usually with low fixed costs and low operational leverage) are particularly sensitive to changes in discount rates.
- Companies in distressed scenarios, operating in stagnant markets and going concern, will be hyposensitive to changes in the prevailing discount rates.

The above are broad generalisation that must be caveated and nuanced before drawing specific conclusion for individual cases.

A second application of sensitivity analysis has to do with looking at a company's leverage in relation to the relative sensitivity of its assets and liabilities. (we have previously looked at how leverage can transfer the sensitivity of operating cash flows onto the sensitivity of equity)

The following matrix shows different potential company sensitivity profiles given leverage:

GRAPH 44



From graph 44 we learn that:

- Companies with the highest debt do not necessarily present the highest Sensitivity to changes in the discount rate. Duration of assets is sometimes more important than debt absolute levels. A company with high asset duration (i.e. companies in infrastructure development), and small levels of debt can be more sensitive to changes in the discount rate than a company with large levels of debt but with cash flows with lower duration.
- Debt can even help reduce interest rate risk if the duration of the liabilities taken on is significantly higher than that of their respective assets. Continuing with the example of companies in a crisis scenario in a low expected growth environment (low asset duration), that refinance their delicate situation with fixed interest long-term loans (high debt duration) can effectively hedge their equity exposure to changes in the discount rate<sup>12</sup>.

<sup>12</sup> This analysis does not hold true if financing happens through variable rates as these have low durations which translate into increase equity exposure rather than mitigation.

## SUMMARY

Sensitivity analysis to changes in interest rates (often in the form of duration analysis) is the basis for fixed income portfolio management; however, a similar exercise analysing the reactivity of the present value of assets to changes in the applied discount rate is seldom used in equity analysis.

Sensitivity (or adjusted duration) is the first derivative of duration with respect to the discount rate and thus represents a linear approximation of an asset's present value reaction to changes in the discount rate. Similarly, duration measures that same reactivity by benchmarking the instrument under study to an equivalent single cash flow asset in number of years. (i.e. an asset with a Duration equal to 7 years is as sensitive to changes in its discount rate as a 7 year zero coupon bond or equivalent single cash flow instrument). Duration, in the same way as sensitivity, allows for enhanced comparability across instruments, in particular with regard to their exposure to interest rate (or discount rate) risk.

As with any derivative, the smaller the changes in the discount rate, the more accurate our estimate will be. This is particularly true in the case of discount rate sensitivity analysis, as it also presents elements of convexity. (the relationship between present value and discount rate is not linear but polynomial). Therefore, it follows reason that the error factor in our calculations will be smallest when changes in the discount rate are small and convexity is low. Alternatively, a more complex formulation accounting for convexity will be required in order to minimise the error factor in our estimations.

Delving deeper into the factors that determine an asset's value sensitivity to changes in the discount rate, we conclude that the key drivers of sensitivity and duration are the required rate of return and the expected growth rate. With the former exhibiting an inverse correlation and the latter a direct one.

Therefore, as the required rate of return decreases, the value of future cash flows increases, with those farther into the future increasing exponentially (this is due to the compounding effect and is the basis for financial discounting) which leads to a greater contribution by those distant cash flows to the present value of the asset and thus to a larger duration. We then conclude that the required rate of return must be inversely correlated with duration and sensitivity. As outlined in this chapter, the convexity effect also means that companies with low risk will be more sensitive to changes in the required rate of return than those companies which face higher uncertainty within their cost of capital.

Conversely, and following the same underlying principle, the greater the expected growth rate, the greater the contribution of distant future cash flows to present value and, consequently, the higher the sensitivity of that company's present value to changes in the discount rate. Therefore, companies with higher growth expectations will suffer from higher sensitivity.

Since the P/E ratio is affected by both by the required rate of return and the expected growth rate, it can be used to estimate sensitivity. Consequently, companies with a high P/E ratio (irrespective of whether due to low required rate of return or high growth) will have higher sensitivity than their low P/E ratio counterparts.

Lastly, another important aspect to take into account is that equity sensitivity is a function of the sensitivities of a company's underlying assets and liabilities. Because of it, any discrepancies between the two ( $D^*_a - D^*_d$ ), will directly impact the sensitivity of the equity ( $D^*_e$ ). Companies holding liabilities that are more sensitive to changes in the interest rate than their respective assets will have the effect of cushioning changes on equity (lower  $D^*_e$ ) meanwhile companies where asset sensitivity is higher than liability sensitivity will directly transfer the impact of that spread, magnifying the sensitivity of their equity to changes in the discount rate (higher  $D^*_e$ ).

Xavier Adserà  
Founding Partner  
ADEQUITA  
[www.adequita.co.uk](http://www.adequita.co.uk)